Name ______________________ Class ____________ Date ____________

6-1 Reteaching

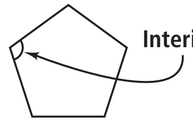
The Polygon Angle-Sum Theorems

Interior Angles of a Polygon

The angles on the inside of a polygon are called *interior angles.*

Polygon Angle-Sum Theorem:

The sum of the measures of the angles of an n-gon is $(n - 2)180$.



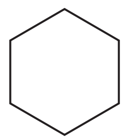
A pentagon has 5 interior angles.

You can write this as a formula. This formula works for regular and irregular polygons.

Sum of angle measures $= (n - 2)180$

Problem

What is the sum of the measures of the angles in a hexagon?



There are six sides, so $n = 6$.

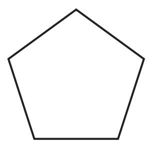
Sum of angle measures	$= (n - 2)180$	
	$= (6 - 2)180$	Substitute 6 for n.
	$= 4(180)$	Subtract.
	$= 720$	Multiply.

The sum of the measures of the angles in a hexagon is 720.

You can use the formula to find the measure of one interior angle of a regular polygon if you know the number of sides.

Problem

What is the measure of each angle in a regular pentagon?



Sum of angle measures	$= (n - 2)180$	
	$= (5 - 2)180$	Substitute 5 for n.
	$= 3(180)$	Subtract.
	$= 540$	Multiply.

Divide by the number of angles:

Measure of each angle	$= 540 \div 5$	
	$= 108$	Divide.

Each angle of a regular pentagon measures 108.

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6-1 Reteaching (continued)

The Polygon Angle-Sum Theorems

Exercises

Find the sum of the interior angles of each polygon.

1. quadrilateral **360** **2.** octagon **1080** **3.** 18-gon **2880**

4. decagon **1440** **5.** 12-gon **1800** **6.** 28-gon **4680**

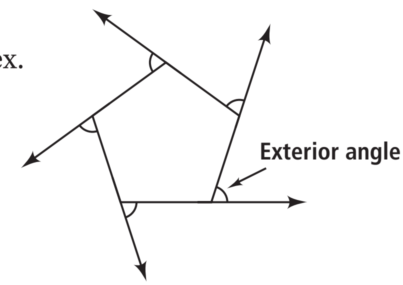
Find the measure of an interior angle of each regular polygon. Round to the nearest tenth if necessary.

7. decagon **144** **8.** 12-gon **150** **9.** 16-gon **157.5**

10. 24-gon **165** **11.** 32-gon **168.8** **12.** 90-gon **176**

Exterior Angles of a Polygon

The exterior angles of a polygon are those formed by extending sides. There is one exterior angle at each vertex.



Polygon Exterior Angle-Sum Theorem:

The sum of the measures of the exterior angles of a polygon is 360.

A pentagon has five exterior angles. The sum of the measures of the exterior angles is always 360, so each exterior angle of a regular pentagon measures 72.

Exercises

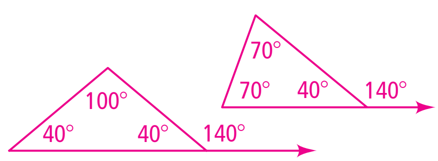
Find the measure of an exterior angle for each regular polygon. Round to the nearest tenth if necessary.

13. octagon **45** **14.** 24-gon **15** **15.** 34-gon **10.6**

16. decagon **36** **17.** heptagon **51.4** **18.** hexagon **60**

19. 30-gon **12** **20.** 28-gon **12.9** **21.** 36-gon **10**

22. Draw a Diagram A triangle has two congruent angles, and an exterior angle that measures 140. Find two possible sets of angle measures for the triangle. Draw a diagram for each. **40, 40, 100; 40, 70, 70**



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6-2 Reteaching

Properties of Parallelograms

Parallelograms

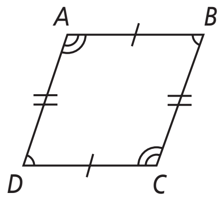
Remember, a *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Here are some attributes of a parallelogram:

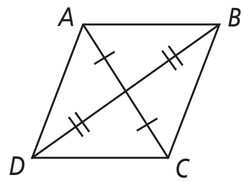
The opposite sides are congruent.

The consecutive angles are supplementary.

The opposite angles are congruent.

The diagonals bisect each other.

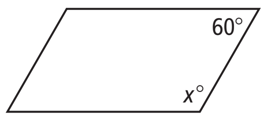




You can use these attributes to solve problems about parallelograms.

Problem

Find the value of x.

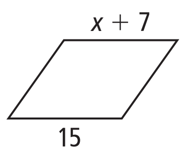


Because the consecutive angles are supplementary,

$$x + 60 = 180$$
$$x = 120$$

Problem

Find the value of x.

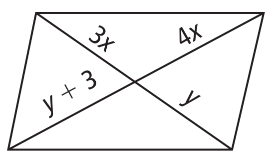


Because opposite sides are congruent,

$$x + 7 = 15$$
$$x = 8$$

Problem

Find the value of x and y.



Because the diagonals bisect each other, $y = 3x$ and $4x = y + 3$.

$4x = y + 3$	
$4x = 3x + 3$	Substitute for *y*.
$x = 3$	Subtraction Property of =
$y = 3x$	Given
$y = 3(3)$	Substitute for *x*.
$y = 9$	Simplify.

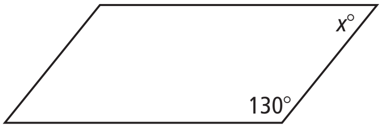
Name ______________________ Class __________ Date __________

6-2 Reteaching (continued)

Properties of Parallelograms

Exercises

Find the value of *x* in each parallelogram.

1. 

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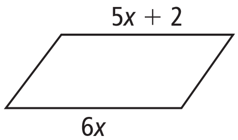
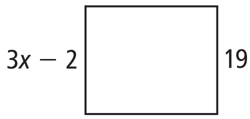
2. 

2

3. **35** 

4. 

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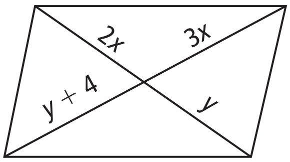
5. 

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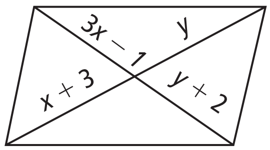
6. 

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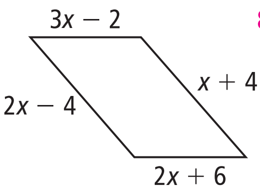
7. 

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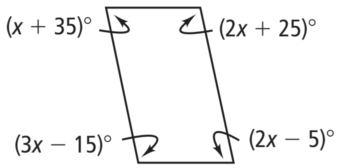
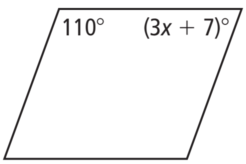
8. 

40

9. 

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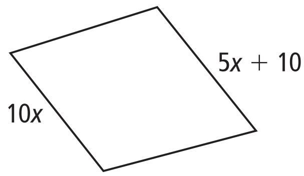
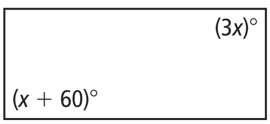
10. 

2

11. 

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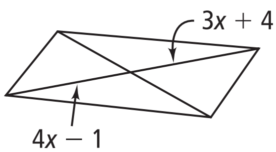
12. 

5

13. **Writing** Write a statement about the consecutive angles of a parallelogram.
Consecutive angles of a parallelogram are supplementary.

14. **Writing** Write a statement about the opposite angles of a parallelogram.
Opposite angles of a parallelogram are congruent.

15. **Reasoning** One angle of a parallelogram is 47. What are the measures of the other three angles in the parallelogram?
47, 133, and 133

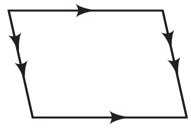
6-3 Reteaching

Proving That a Quadrilateral Is a Parallelogram

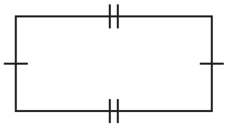
Is a quadrilateral a parallelogram?

There are five ways that you can confirm that a quadrilateral is a parallelogram.

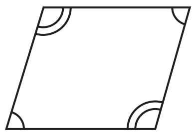
If both pairs of opposite sides are parallel, then the quadrilateral is a parallelogram.



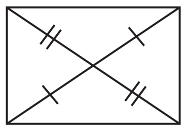
If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.



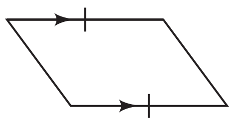
If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.



If the diagonals bisect each other, then the quadrilateral is a parallelogram.



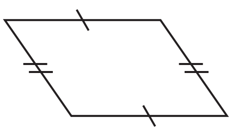
If one pair of sides is both congruent and parallel, then the quadrilateral is a parallelogram.



Exercises

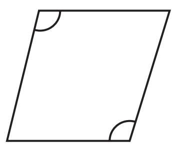
Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

1.



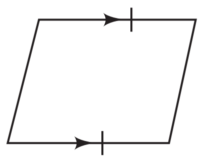
yes; opposite sides $\cong$

2.



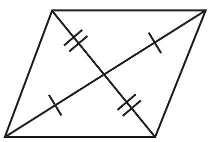
no; not enough info

3.



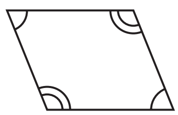
yes; 1 pair of sides $\cong$ and $\parallel$

4.



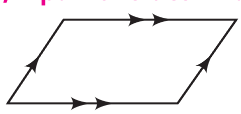
Yes; diagonals bisect each other.

5.



yes; opposite $\angle$s $\cong$

6.

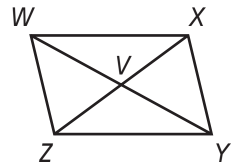


yes; opposite sides $\parallel$

6-3 Reteaching (continued)

Proving That a Quadrilateral Is a Parallelogram

Determine whether the given information is sufficient to prove that quadrilateral *WXYZ* is a parallelogram.



7. $\overline{WY}$ bisects $\overline{ZX}$ no

8. $\overline{WX} \parallel \overline{ZY}$; $\overline{WZ} \cong \overline{XY}$ no

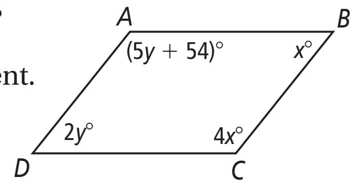
9. $\overline{VZ} \cong \overline{VX}$; $\overline{WX} \cong \overline{YZ}$ no

10. $\angle VWZ \cong \angle VYX$; $\overline{WZ} \cong \overline{XY}$ yes

You can also use the requirements for a parallelogram to solve problems.

Problem

For what value of x and y must figure *ABCD* be a parallelogram?



In a parallelogram, the two pairs of opposite angles are congruent. So, in *ABCD*, you know that $x = 2y$ and $5y + 54 = 4x$. You can use these two expressions to solve for x and y.

Step 1: Solve for y.

$5y + 54 = 4x$

$5y + 54 = 4(2y)$ Substitute $2y$ for x.

$5y + 54 = 8y$ Simplify.

$54 = 3y$ Subtract $5y$ from each side.

$18 = y$ Divide each side by 3.

Step 2: Solve for x.

$x = 2y$ Opposite angles of a parallelogram are congruent.

$x = 2(18)$ Substitute 18 for y.

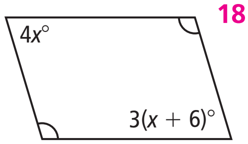
$x = 36$ Simplify.

For *ABCD* to be a parallelogram, x must be 36 and y must be 18.

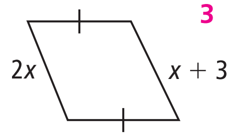
Exercises

For what value of x must the quadrilateral be a parallelogram?

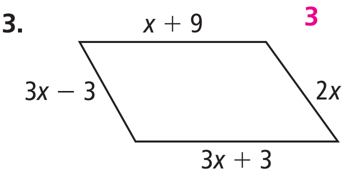
11. 18



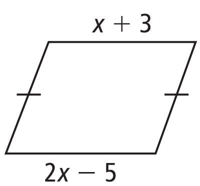
12. 3



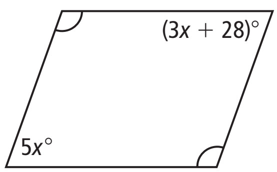
13. 3



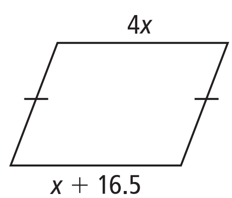
14. 8



15. 14



16. 5.5



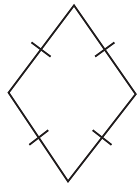
Name ______________________ Class ____________ Date ____________

6-4 Reteaching

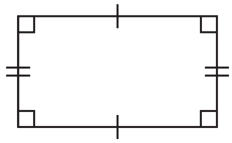
Properties of Rhombuses, Rectangles, and Squares

Rhombuses, rectangles, and *squares* share some characteristics. But they also have some unique features.

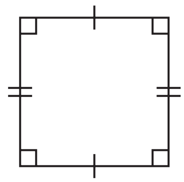
A rhombus is a parallelogram with four congruent sides.



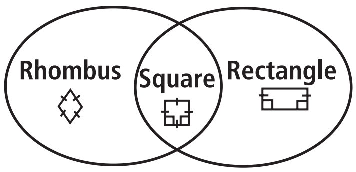
A rectangle is a parallelogram with four congruent angles. These angles are all right angles.



A square is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus. A square is the only type of rectangle that can also be a rhombus.



Here is a Venn diagram to help you see the relationships.



There are some special features for each type of figure.

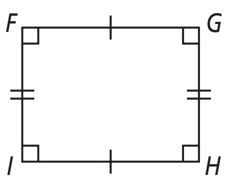
Rhombus: The diagonals are perpendicular.
The diagonals bisect a pair of opposite angles.

Rectangles: The diagonals are congruent.

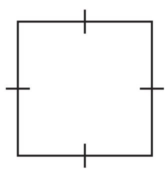
Squares: The diagonals are perpendicular.
The diagonals bisect a pair of opposite angles (forming two 45° angles at each vertex).
The diagonals are congruent.

Exercises

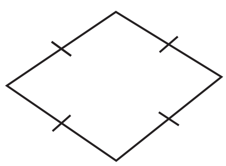
Decide whether the parallelogram is a rhombus, a rectangle, or a square.

1. 

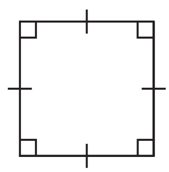
rectangle

2. 

rhombus

3. 

rhombus

4. 

square

6-4 Reteaching (continued)

Properties of Rhombuses, Rectangles, and Squares

List the quadrilaterals that have the given property. Choose among *parallelogram, rhombus, rectangle,* and *square.*

5. Opposite angles are supplementary.
rectangle, square

6. Consecutive sides are ≅.
rhombus, square

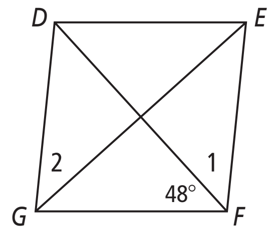
7. Consecutive sides are ⊥.
rectangle, square

8. Consecutive angles are ≅.
rectangle, square

You can use the properties of rhombuses, rectangles, and squares to solve problems.

Problem

Determine the measure of the numbered angles in rhombus *DEFG*.



$\angle 1$ is part of a bisected angle. $m\angle DFG = 48$, so $m\angle 1 = 48$.

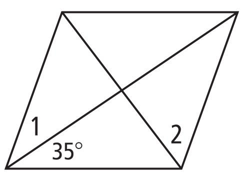
Consecutive angles of a parallelogram are supplementary.
$m\angle EFG = 48 + 48 = 96$, so $m\angle DGF = 180 - 96 = 84$.

The diagonals bisect the vertex angle, so $m\angle 2 = 84 \div 2 = 42$.

Exercises

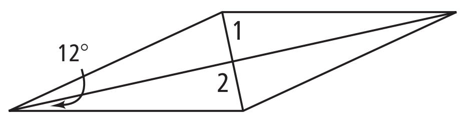
Determine the measure of the numbered angles in each rhombus.

9.



35; 55

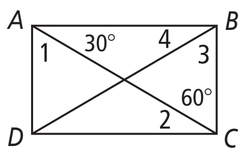
10.



78; 90

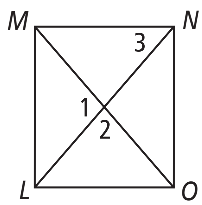
Determine the measure of the numbered angles in each figure.

11. rectangle *ABCD*



60; 30; 60; 30

12. square *LMNO*



90; 90; 45

Algebra ***TUVW*** **is a rectangle. Find the value of *x* and the length of each diagonal.**

13. $TV = 3x$ and $UW = 5x - 10$
5; 15; 15

14. $TV = 2x - 4$ and $UW = x + 10$
14; 24; 24

15. $TV = 6x + 4$ and $UW = 4x + 8$
2; 16; 16

16. $TV = 7x + 6$ and $UW = 9x - 18$
12; 90; 90

17. $TV = 8x - 2$ and $UW = 5x + 7$
3; 22; 22

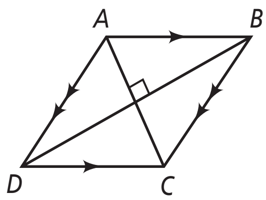
18. $TV = 10x - 4$ and $UW = 3x + 24$
4; 36; 36

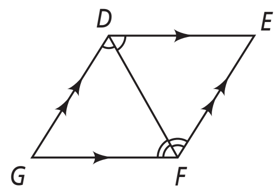
Name ______________________ Class ____________ Date ____________

6-5 Reteaching

Conditions for Rhombuses, Rectangles, and Squares

A parallelogram is a rhombus if either of these conditions is met:



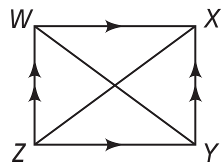


1) The diagonals of the parallelogram are perpendicular. (Theorem 6-16)

2) A diagonal of the parallelogram bisects a pair of opposite angles. (Theorem 6-17)

A parallelogram is a rectangle if the diagonals of the parallelogram are congruent.

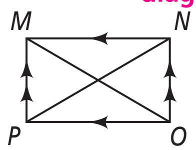
$\overline{WY} \cong \overline{XZ}$



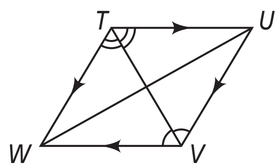
Exercises

Classify each of the following parallelograms as a *rhombus,* a *rectangle,* or a *square.* For each, explain.

1. $\overline{MO} \cong \overline{PN}$ **Rectangle; the diagonals are ≅.**

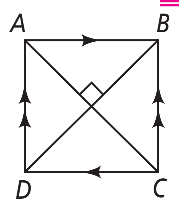


2.



Rhombus; the diagonals bisect opposite angles.

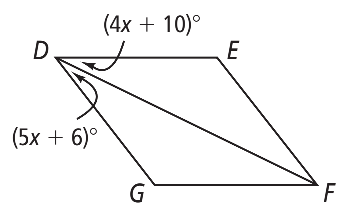
3. $\overline{AC} \cong \overline{BD}$ **Square; the diagonals are ≅ and ⊥.**



Use the properties of rhombuses and rectangles to solve problems.

Problem

For what value of x is ▱$DEFG$ a rhombus?



In a rhombus, diagonals bisect opposite angles.

So, $m\angle GDF = m\angle EDF$.

$(4x + 10) = (5x + 6)$	Set angle measures equal to each other.
$10 = x + 6$	Subtract 4x from each side.
$4 = x$	Subtract 6 from each side.

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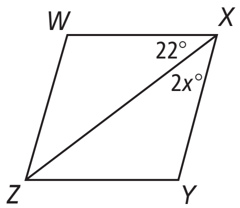
6-5 Reteaching (continued)

Conditions for Rhombuses, Rectangles, and Squares

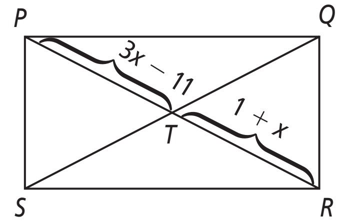
Exercises

4. For what value of x is ▱$WXYZ$ a rhombus? **11**

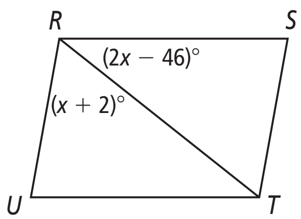


5. $SQ = 14$. For what value of x is ▱$PQRS$ a rectangle?

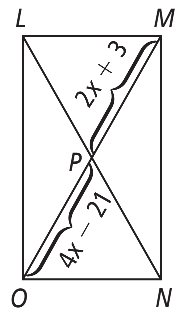
Solve for PT. Solve for PR. **6; 7; 14**



6. For what value of x is ▱$RSTU$ a rhombus? What is $m\angle SRT$? What is $m\angle URS$? **48; 50; 100**



7. $LN = 54$. For what value of x is ▱$LMNO$ a rectangle? **12**



8. Given: ▱$ABCD$, $\overline{AC} \perp \overline{BD}$ at E.

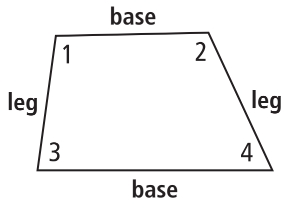
Prove: $ABCD$ is a rhombus.

Statements	Reasons
1) $\overline{AE} \cong \overline{CE}$	1) __?__ **Diagonals of a ▱ bisect each other.**
2) $\overline{AC} \perp \overline{BD}$ at E	2) __?__ **Given**
3) __?__ **$\angle AED$ and $\angle CED$ are right angles.**	3) Definition of perpendicular lines
4) __?__ **$\angle AED \cong \angle CED$**	4) __?__ **All right angles are congruent.**
5) __?__ **$\overline{DE} \cong \overline{DE}$**	5) Reflexive Property of Congruence
6) $\triangle AED \cong \triangle CED$	6) __?__ **SAS Postulate**
7) $\overline{AD} \cong \overline{CD}$	7) __?__ **CPCTC**
8) __?__ **$\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$**	8) Opposite sides of a ▱ are ≅.
9) __?__ **$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$**	9) __?__ **Transitive Property of Congruence**
10) $ABCD$ is a rhombus.	10) __?__ **Definition of rhombus**

6-6 Reteaching

Trapezoids and Kites

A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. The two parallel sides are called *bases.* The two nonparallel sides are called *legs.*

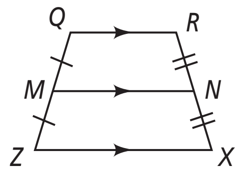


A pair of base angles share a common base.

$\angle 1$ and $\angle 2$ are one pair of base angles.

$\angle 3$ and $\angle 4$ are a second pair of base angles.

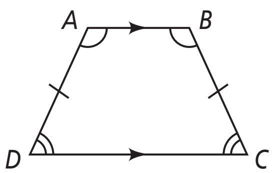
In any trapezoid, the *midsegment* is parallel to the bases. The length of the midsegment is half the sum of the lengths of the bases.



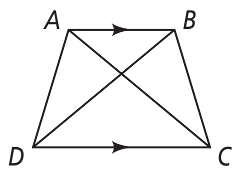
$$MN = \frac{1}{2}(QR + ZX)$$

An *isosceles trapezoid* is a trapezoid in which the legs are congruent. An isosceles trapezoid has some special properties:

Each pair of base angles is congruent.



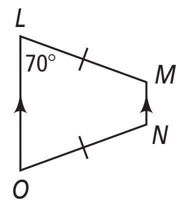
The diagonals are congruent.



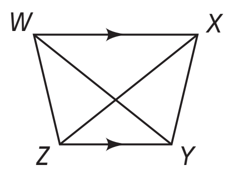
$AC = BD$

Exercises

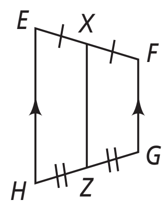
1. In trapezoid *LMNO,* what is the measure of $\angle OLM$? **70**
What is the measure of $\angle LMN$? **110**



2. *WXYZ* is an isosceles trapezoid and $WY = 12$. What is *XZ?* **12**



3. $\overline{XZ}$ is the midsegment of trapezoid *EFGH.* If $FG = 8$ and $EH = 12$, what is *XZ?* **10**



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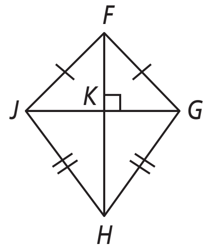
6-6 Reteaching (continued)

Trapezoids and Kites

A *kite* is a quadrilateral in which two pairs of consecutive sides are congruent and no opposite sides are congruent.

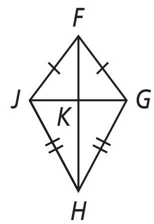


In a kite, the diagonals are perpendicular. The diagonals look like the crossbars in the frame of a typical kite that you fly.

Notice that the sides of a kite are the hypotenuses of four right triangles whose legs are formed by the diagonals.

Problem

Write a two-column proof to identify three pairs of congruent triangles in kite *FGHJ*.



Statements	Reasons
1) $m\angle FKG = m\angle GKH = m\angle HKJ = m\angle JKF = 90$	1) Theorem 6-22
2) $\overline{FG} \cong \overline{FJ}$	2) Given
3) $\overline{FK} \cong \overline{FK}$	3) Reflexive Property of Congruence
4) $\triangle FKG \cong \triangle FKJ$	4) HL Theorem
5) $\overline{JK} \cong \overline{KG}$	5) CPCTC
6) $\overline{KH} \cong \overline{KH}$	6) Reflexive Property of Congruence
7) $\triangle JKH \cong \triangle GKH$	7) SAS Postulate
8) $\overline{JH} \cong \overline{GH}$	8) Given
9) $\overline{FH} \cong \overline{FH}$	9) Reflexive Property of Congruence
10) $\triangle FJH \cong \triangle FGH$	10) SSS Postulate

So $\triangle FKG \cong \triangle FKJ$, $\triangle JKH \cong \triangle GKH$, and $\triangle FJH \cong \triangle FGH$.

Exercises

In kite *FGHJ* in the problem, $m\angle JFK = 38$ and $m\angle KGH = 63$. Find the following angle and side measures.

4. $m\angle FKJ$ 90

5. $m\angle FJK$ 52

6. $m\angle FKG$ 90

7. $m\angle KFG$ 38

8. $m\angle FGK$ 52

9. $m\angle GKH$ 90

10. $m\angle KHG$ 27

11. $m\angle KJH$ 63

12. $m\angle JHK$ 27

13. If $FG = 4.25$, what is *JF*? 4.25

14. If $HG = 5$, what is *JH*? 5

15. If $JK = 8.5$, what is *GJ*? 17