

4-1 Reteaching

Congruent Figures

Given $ABCD \cong QRST$, find corresponding parts using the names. Order matters.

For example, $\overset{\frown}{A}$ $ABCD$ This shows that $\angle A$ corresponds to $\angle Q$.
 $\overset{\frown}{Q}$ $QRST$ Therefore, $\angle A \cong \angle Q$.

For example, \overline{BC} $ABCD$ This shows that \overline{BC} corresponds to \overline{RS} .
 \overline{RS} $QRST$ Therefore, $\overline{BC} \cong \overline{RS}$.

Exercises

Find corresponding parts using the order of the letters in the names.

- Identify the remaining three pairs of corresponding angles and sides between $ABCD$ and $QRST$ using the circle technique shown above.

$\angle B \cong \angle R$, $\angle C \cong \angle S$, $\angle D \cong \angle T$, $\overline{AB} \cong \overline{QR}$, $\overline{CD} \cong \overline{ST}$, and $\overline{DA} \cong \overline{TQ}$

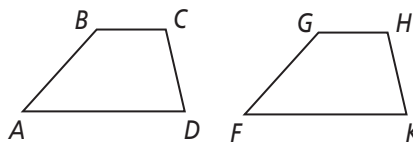
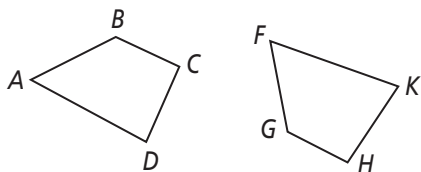
Angles: $ABCD$ $ABCD$ $ABCD$ Sides: $ABCD$ $ABCD$ $ABCD$
 $QRST$ $QRST$ $QRST$ $QRST$ $QRST$ $QRST$

- Which pair of corresponding sides is hardest to identify using this technique?

Answers may vary. Sample: \overline{AD} and \overline{QT}

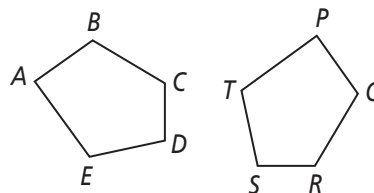
Find corresponding parts by redrawing figures.

- The two congruent figures below at the left have been redrawn at the right. Why are the corresponding parts easier to identify in the drawing at the right?



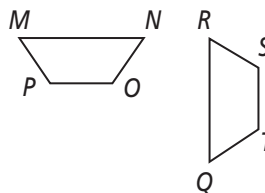
Answers may vary. Sample: The drawing at the right shows figures in same orientation.

- Redraw the congruent polygons at the right in the same orientation. Identify all pairs of corresponding sides and angles. **Check students' work.** $\angle A$ and $\angle P$, $\angle B$ and $\angle Q$, $\angle C$ and $\angle R$, $\angle D$ and $\angle S$, $\angle E$ and $\angle T$, \overline{AB} and \overline{PQ} , \overline{BC} and \overline{QR} , \overline{CD} and \overline{RS} , \overline{DE} and \overline{ST} , and \overline{EA} and \overline{TP} all correspond.



- $MNOP \cong QRST$. Identify all pairs of congruent sides and angles.

$\angle M \cong \angle Q$, $\angle N \cong \angle R$, $\angle O \cong \angle S$, $\angle P \cong \angle T$,
 $\overline{MN} \cong \overline{QR}$, $\overline{NO} \cong \overline{RS}$, $\overline{OP} \cong \overline{ST}$, and $\overline{PM} \cong \overline{TQ}$



4-1 Reteaching (continued)

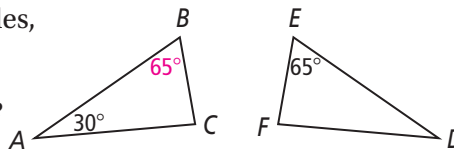
Congruent Figures

Problem

Given $\triangle ABC \cong \triangle DEF$, $m\angle A = 30$, and $m\angle E = 65$, what is $m\angle C$?

How might you solve this problem? Sketch both triangles, and put all the information on both diagrams.

$m\angle A = 30$; therefore, $m\angle D = 30$. How do you know?
Because $\angle A$ and $\angle D$ are corresponding parts of congruent triangles.



Exercises

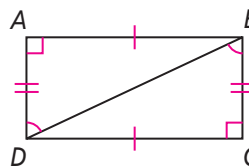
Work through the exercises below to solve the problem above.

- What angle in $\triangle ABC$ has the same measure as $\angle E$? What is the measure of that angle? Add the information to your sketch of $\triangle ABC$.
 $\angle B$; 65
- You know the measures of two angles in $\triangle ABC$. How can you find the measure of the third angle?
Answers may vary. Sample: Use Triangle Angle-Sum Thm. Set sum of all three angles equal to 180.
- What is $m\angle C$? How did you find your answer?
85; answers may vary. Sample: $m\angle C = 180 - (60 + 35)$

Before writing a proof, add the information implied by each given statement to your sketch. Then use your sketch to help you with Exercises 9–12.

Add the information implied by each given statement.

- Given: $\angle A$ and $\angle C$ are right angles.
 $m\angle A = m\angle C = 90$, $\overline{DA} \perp \overline{AB}$ and $\overline{DC} \perp \overline{BC}$
- Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$.
 $ABCD$ is a parallelogram because it has opposite sides that are congruent.
- Given: $\angle ADB \cong \angle CBD$.
 $\overline{AD} \parallel \overline{BC}$
- Can you conclude that $\angle ABD \cong \angle CDB$ using the given information above? If so, how?
Yes; use the Third Angles Thm.
- How can you conclude that the third side of both triangles is congruent?
The third side is shared by both triangles and is congruent by the Refl. Prop. of Congruence.



4-2 Reteaching

Triangle Congruence by SSS and SAS

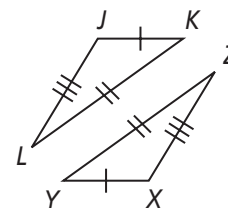
You can prove that triangles are congruent using the two postulates below.

Postulate 4-1: Side-Side-Side (SSS) Postulate

If all three sides of a triangle are congruent to all three sides of another triangle, then those two triangles are congruent.

If $\overline{JK} \cong \overline{XY}$, $\overline{KL} \cong \overline{YZ}$, and $\overline{JL} \cong \overline{XZ}$, then $\triangle JKL \cong \triangle XYZ$.

In a triangle, the angle formed by any two sides is called the *included angle* for those sides.

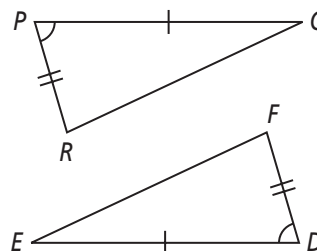


Postulate 4-2: Side-Angle-Side (SAS) Postulate

If two sides and the included angle of a triangle are congruent to two sides and the included angle of another triangle, then those two triangles are congruent.

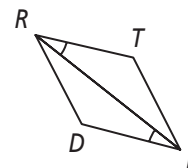
If $\overline{PQ} \cong \overline{DE}$, $\overline{PR} \cong \overline{DF}$, and $\angle P \cong \angle D$, then $\triangle PQR \cong \triangle DEF$.

$\angle P$ is included by \overline{QP} and \overline{PR} . $\angle D$ is included by \overline{ED} and \overline{DF} .

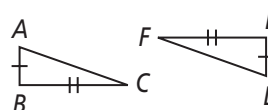


Exercises

- What other information do you need to prove $\triangle TRF \cong \triangle DFR$ by SAS? Explain. **$\overline{DF} \cong \overline{TR}$; by the Reflexive Property of Congruence, $\overline{RF} \cong \overline{FR}$. It is given that $\angle TRF \cong \angle DFR$. These are the included angles for the corresponding congruent sides.**



- What other information do you need to prove $\triangle ABC \cong \triangle DEF$ by SAS? Explain. **$\angle B \cong \angle E$; These are the included angles between the corresponding congruent sides.**

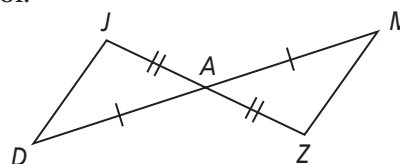


- Developing Proof** Copy and complete the flow proof.

Given: $\overline{DA} \cong \overline{MA}$, $\overline{AJ} \cong \overline{AZ}$

Prove: $\triangle JDA \cong \triangle ZMA$

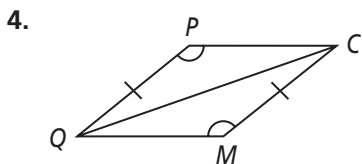
$\overline{DA} \cong \overline{MA}$		
<div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">?</div>	<div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">?</div>	
Given		
<div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">$\overline{AJ} \cong \overline{AZ}$</div>	<div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">?</div>	
? Given		
<div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">?</div>	<div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">?</div>	<div style="border: 1px solid black; padding: 5px; width: 60px; margin: 0 auto;">?</div>
Vertical \triangle are \cong .	$\angle JAD \cong \angle ZAM$? SAS



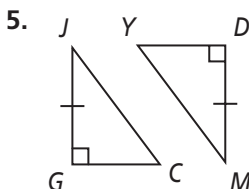
4-2 Reteaching (continued)

Triangle Congruence by SSS and SAS

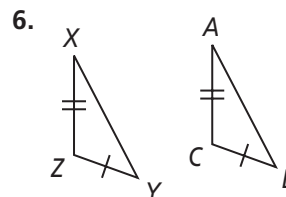
Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information*. Explain your answer.



Not enough information; two pairs of corresponding sides are congruent, but the congruent angles are not the included angles.



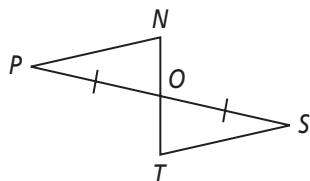
Not enough information; you need to know if $\overline{GC} \cong \overline{DY}$.



Not enough information; only two pairs of corresponding sides are congruent. You need to know if $\overline{AB} \cong \overline{XY}$ or $\angle Z \cong \angle C$.

7. Given: $\overline{PO} \cong \overline{SO}$, O is the midpoint of \overline{NT} .

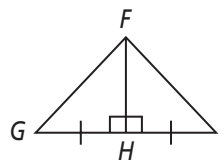
Prove: $\triangle NOP \cong \triangle TOS$



Statements: 1) $\overline{PO} \cong \overline{SO}$; 2) O is the midpoint of \overline{NT} ; 3) $\overline{NO} \cong \overline{TO}$; 4) $\angle NOP \cong \angle TOS$; 5) $\triangle NOP \cong \triangle TOS$;
Reasons: 1) Given; 2) Given; 3) Def. of midpoint; 4) Vert. \angle are \cong ; 5) SAS

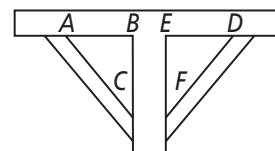
8. Given: $\overline{HI} \cong \overline{HG}$, $\overline{FH} \perp \overline{GI}$

Prove: $\triangle FHI \cong \triangle FHG$



Statements: 1) $\overline{FH} \cong \overline{FH}$; 2) $\overline{HI} \cong \overline{HG}$, $\overline{FH} \perp \overline{GI}$; 3) $\angle FHG$ and $\angle FHI$ are rt. \angle s; 4) $\angle FHG \cong \angle FHI$; 5) $\triangle FHI \cong \triangle FHG$;
Reasons: 1) Refl. Prop.; 2) Given; 3) Def. of perpendicular; 4) All rt. \angle are \cong ; 5) SAS

9. A carpenter is building a support for a bird feeder. He wants the triangles on either side of the vertical post to be congruent. He measures and finds that $\overline{AB} \cong \overline{DE}$ and that $\overline{AC} \cong \overline{DF}$. What would he need to measure to prove that the triangles are congruent using SAS? What would he need to measure to prove that they are congruent using SSS?



For SAS, he would need to determine if $\angle BAC \cong \angle EDF$; for SSS, he would need to determine if $\overline{BC} \cong \overline{EF}$.

10. An artist is drawing two triangles. She draws each so that two sides are 4 in. and 5 in. long and an angle is 55° . Are her triangles congruent? Explain.

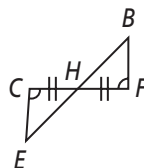
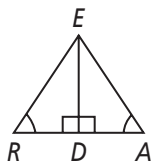
Answers may vary. Sample: Maybe; if both the 55° angles are between the 4-in. and 5-in. sides, then the triangles are congruent by SAS.

4-3 Reteaching

Triangle Congruence by ASA and AAS

Problem

Can the ASA Postulate or the AAS Theorem be applied directly to prove the triangles congruent?

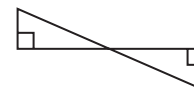
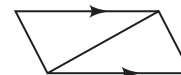


- a. Because $\angle RDE$ and $\angle ADE$ are right angles, they are congruent. $\overline{ED} \cong \overline{ED}$ by the Reflexive Property of \cong , and it is given that $\angle R \cong \angle A$. Therefore, $\triangle RDE \cong \triangle ADE$ by the AAS Theorem.
- b. It is given that $\overline{CH} \cong \overline{FH}$ and $\angle C \cong \angle F$. Because $\angle CHE$ and $\angle FHB$ are vertical angles, they are congruent. Therefore, $\triangle CHE \cong \triangle FHB$ by the ASA Postulate.

Exercises

Indicate congruences.

- Copy the top figure at the right. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the ASA Postulate.
- Copy the second figure shown. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the AAS Theorem.
- Draw and mark two triangles that are congruent by either the ASA Postulate or the AAS Theorem. **Check students' work.**



What additional information would you need to prove each pair of triangles congruent by the stated postulate or theorem?

4. ASA Postulate $\angle ABD \cong \angle CBD$

5. AAS Theorem $\angle JMK \cong \angle LKM$, $\angle JKM \cong \angle LMK$, $\angle JMK \cong \angle LMK$, or $\angle JKM \cong \angle LKM$

6. ASA Postulate $\angle ZXY \cong \angle ZVU$

7. AAS Theorem $\angle Y \cong \angle O$

8. AAS Theorem $\angle P \cong \angle A$

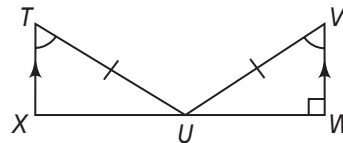
9. ASA Postulate $\angle CYL \cong \angle ALY$

4-3 Reteaching (continued)

Triangle Congruence by ASA and AAS

10. Provide the reason for each step in the two-column proof.

Given: $\overline{TX} \parallel \overline{VW}$, $\overline{TU} \cong \overline{VU}$, $\angle XTU \cong \angle WVU$,
 $\angle UWV$ is a right angle.



Prove: $\triangle TUX \cong \triangle VUW$

Statements	Reasons
1) $\angle UWV$ is a right angle.	1) ? Given
2) $\overline{VW} \perp \overline{XW}$	2) ? Definition of perpendicular lines
3) $\overline{TX} \parallel \overline{VW}$	3) ? Given
4) $\overline{TX} \perp \overline{XW}$	4) ? Perpendicular Transversal Theorem
5) $\angle UXT$ is a right angle.	5) ? Definition of perpendicular lines
6) $\angle UWV \cong \angle UXT$	6) ? All right angles are congruent.
7) $\overline{TU} \cong \overline{VU}$	7) ? Given
8) $\angle XTU \cong \angle WVU$	8) ? Given
9) $\triangle TUX \cong \triangle VUW$	9) ? AAS Theorem

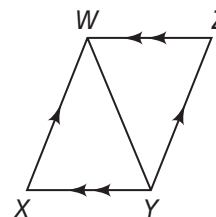
11. Write a paragraph proof.

Given: $\overline{WX} \parallel \overline{ZY}$; $\overline{WZ} \parallel \overline{XY}$

Prove: $\triangle WXY \cong \triangle YZW$

It is given that $\overline{WX} \parallel \overline{ZY}$ and $\overline{WZ} \parallel \overline{XY}$, so $\angle XWY \cong \angle ZYW$ and $\angle XYW \cong \angle ZWY$, by the Alternate Interior \sphericalangle Thm.

$\overline{WY} \cong \overline{YW}$ by the Reflexive Property of \cong . So, by ASA Post. $\triangle WXY \cong \triangle YZW$.



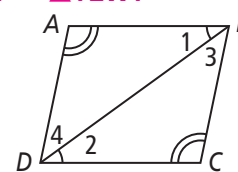
12. **Developing Proof** Complete the proof by filling in the blanks.

Given: $\angle A \cong \angle C$, $\angle 1 \cong \angle 2$

Prove: $\triangle ABD \cong \triangle CDB$

Proof: $\angle A \cong \angle C$ and $\angle 1 \cong \angle 2$ are given. $\overline{DB} \cong \overline{BD}$ by ?.

So, $\triangle ABD \cong \triangle CDB$ by ? . **AAS**

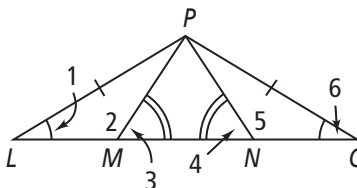


Refl. Prop. of Congruence

13. Write a paragraph proof.

Given: $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 4$, $\overline{LP} \cong \overline{OP}$

Prove: $\triangle LMP \cong \triangle ONP$



$\angle 3 \cong \angle 4$ is given. Therefore, $m\angle 3 = m\angle 4$, by def. of $\cong \sphericalangle$ s. Because $\angle 2$ and $\angle 3$ are linear pairs, and $\angle 4$ and $\angle 5$ are linear pairs, the pairs of angles are suppl. Therefore, $\angle 2 \cong \angle 5$ by the Congruent Suppl. Thm. $\angle 1 \cong \angle 6$ and $\overline{LP} \cong \overline{OP}$ are given, so $\triangle LMP \cong \triangle ONP$, by the AAS Thm.

4-4 Reteaching

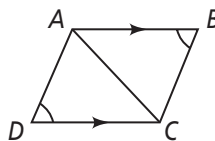
Using Corresponding Parts of Congruent Triangles

If you can show that two triangles are congruent, then you can show that all the corresponding angles and sides of the triangles are congruent.

Problem

Given: $\overline{AB} \parallel \overline{DC}$, $\angle B \cong \angle D$

Prove: $\overline{BC} \cong \overline{DA}$



In this case you know that $\overline{AB} \parallel \overline{DC}$. \overline{AC} forms a transversal and creates a pair of alternate interior angles, $\angle BAC$ and $\angle DCA$.

You have two pairs of congruent angles, $\angle BAC \cong \angle DCA$ and $\angle B \cong \angle D$. Because you know that the shared side is congruent to itself, you can use AAS to show that the triangles are congruent. Then use the fact that corresponding parts are congruent to show that $\overline{BC} \cong \overline{DA}$. Here is the proof:

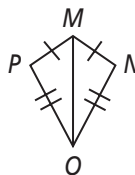
Statements	Reasons
1) $\overline{AB} \parallel \overline{DC}$	1) Given
2) $\angle BAC \cong \angle DCA$	2) Alternate Interior Angles Theorem
3) $\angle B \cong \angle D$	3) Given
4) $\overline{AC} \cong \overline{CA}$	4) Reflexive Property of Congruence
5) $\triangle ABC \cong \triangle CDA$	5) AAS
6) $\overline{BC} \cong \overline{DA}$	6) CPCTC

Exercises

1. Write a two-column proof.

Given: $\overline{MN} \cong \overline{MP}$, $\overline{NO} \cong \overline{PO}$

Prove: $\angle N \cong \angle P$



Statements	Reasons
1) ? $\overline{MN} \cong \overline{MP}$, $\overline{NO} \cong \overline{PO}$	1) Given
2) $\overline{MO} \cong \overline{MO}$	2) ? Reflexive Property of \cong
3) ? $\triangle MNO \cong \triangle MPO$	3) ? SSS
4) $\angle N \cong \angle P$	4) ? CPCTC

4-4 Reteaching (continued)

Using Corresponding Parts of Congruent Triangles

2. Write a two-column proof.

Given: \overline{PT} is a median and an altitude of $\triangle PRS$.

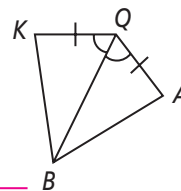
Prove: \overline{PT} bisects $\angle RPS$.

Statements	Reasons
1) \overline{PT} is a median of $\triangle PRS$.	1) ? Given
2) ? T is the midpoint of \overline{RS}.	2) Definition of median
3) ? $\overline{RT} \cong \overline{ST}$	3) Definition of midpoint
4) \overline{PT} is an altitude of $\triangle PRS$.	4) ? Given
5) $\overline{PT} \perp \overline{RS}$	5) ? Definition of altitude
6) $\angle PTS$ and $\angle PTR$ are right angles.	6) ? Definition of perpendicular
7) ? $\angle PTS \cong \angle PTR$	7) All right angles are congruent.
8) ? $\overline{PT} \cong \overline{PT}$	8) Reflexive Property of Congruence
9) ? $\triangle PTS \cong \triangle PTR$	9) SAS
10) $\angle TPS \cong \angle TPR$	10) ? CPCTC
11) ? \overline{PT} bisects $\angle RPS$.	11) ? Definition of angle bisector

3. Write a two-column proof.

Given: $\overline{QK} \cong \overline{QA}$; \overline{QB} bisects $\angle KQA$.

Prove: $\overline{KB} \cong \overline{AB}$

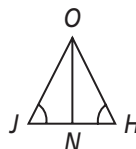


Statements	Reasons
1) $\overline{QK} \cong \overline{QA}$; \overline{QB} bisects $\angle KQA$.	1) Given
2) $\angle KQB \cong \angle AQB$	2) Def. of \angle bis.
3) $\overline{BQ} \cong \overline{BQ}$	3) Refl. Prop. of Congruence
4) $\triangle KBQ \cong \triangle ABQ$	4) SAS
5) $\overline{KB} \cong \overline{AB}$	5) CPCTC

4. Write a two-column proof.

Given: \overline{ON} bisects $\angle JOH$, $\angle J \cong \angle H$

Prove: $\overline{JN} \cong \overline{HN}$



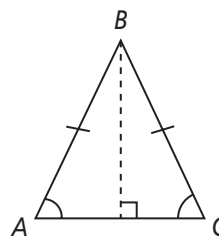
Statements	Reasons
1) \overline{ON} bisects $\angle JOH$, $\angle J \cong \angle H$	1) Given
2) $\angle JON \cong \angle HON$	2) Def. of \angle bis.
3) $\overline{ON} \cong \overline{ON}$	3) Refl. Prop. of Congruence
4) $\triangle JON \cong \triangle HON$	4) AAS
5) $\overline{JN} \cong \overline{HN}$	5) CPCTC

4-5 Reteaching

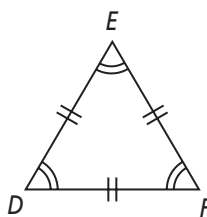
Isosceles and Equilateral Triangles

Two special types of triangles are isosceles triangles and equilateral triangles.

An *isosceles* triangle is a triangle with two congruent sides. The base angles of an isosceles triangle are also congruent. An altitude drawn from the shorter base splits an isosceles triangle into two congruent right triangles.



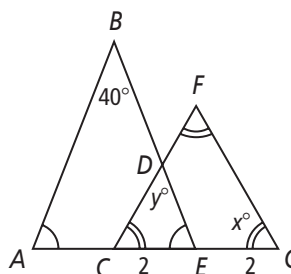
An *equilateral* triangle is a triangle that has three congruent sides and three congruent angles. Each angle measures 60° .



You can use the special properties of isosceles and equilateral triangles to find or prove different information about a given figure.

Look at the figure at the right.

You should be able to see that one of the triangles is equilateral and one is isosceles.



Problem

What is $m\angle A$?

$\triangle ABC$ is isosceles because it has two base angles that are congruent. Because the sum of the measures of the angles of a triangle is 180, and $m\angle B = 40$, you can solve to find $m\angle A$.

$$m\angle A + m\angle B + m\angle BEA = 180$$

There are 180° in a triangle.

$$m\angle A + 40 + m\angle A = 180$$

Substitution Property

$$2m\angle A + 40 = 180$$

Combine like terms.

$$2m\angle A = 140$$

Subtraction Property of Equality

$$m\angle A = 70$$

Division Property of Equality

Problem

What is FC ?

$\triangle CFG$ is equilateral because it has three congruent angles.

$$CG = (2 + 2) = 4, \text{ and } CG = FG = FC.$$

So, $FC = 4$.

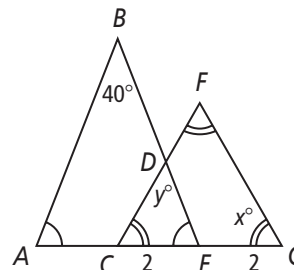
4-5 Reteaching (continued)

Isosceles and Equilateral Triangles

Problem

What is the value of x ?

Because x is the measure of an angle in an equilateral triangle, $x = 60$.



Problem

What is the value of y ?

$$m\angle DCE + m\angle DEC + m\angle EDC = 180$$

$$60 + 70 + y = 180$$

$$y = 50$$

There are 180° in a triangle.

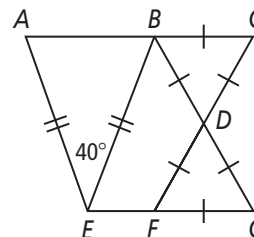
Substitution Property

Subtraction Property of Equality

Exercises

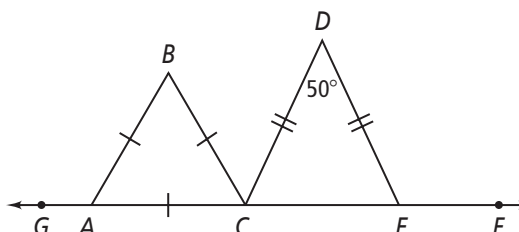
Complete each statement. Explain why it is true.

- $\angle EAB \cong \underline{\quad ? \quad}$
 $\angle EBA$; base angles of an isosceles triangle are congruent.
- $\angle BCD \cong \underline{\quad ? \quad} \cong \angle DBC$
 $\angle CDB$; the angles of an equilateral triangle are congruent.
- $\overline{FG} \cong \underline{\quad ? \quad} \cong \overline{DF}$
 \overline{GD} ; the sides of an equilateral triangle are congruent.



Determine the measure of the indicated angle.

- $\angle ACB$ **60**
- $\angle DCE$ **65**
- $\angle BCD$ **55**



Algebra Find the value of x and y .

- 35; 35**

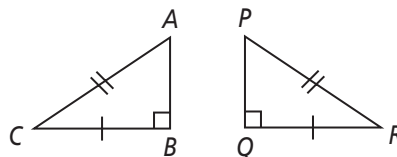
- 65; 50**

- Reasoning** An exterior angle of an isosceles triangle has a measure 140.
Find two possible sets of measures for the angles of the triangle. **40, 40, 100; 40, 70, 70**

4-6 Reteaching

Congruence in Right Triangles

Two right triangles are congruent if they have congruent hypotenuses and if they have one pair of congruent legs. This is the *Hypotenuse-Leg (HL) Theorem*.



$\triangle ABC \cong \triangle PQR$ because they are both right triangles, their hypotenuses are congruent ($\overline{AC} \cong \overline{PR}$), and one pair of legs is congruent ($\overline{BC} \cong \overline{QR}$).

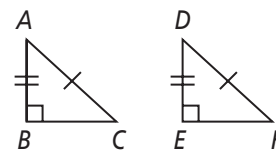
Problem

How can you prove that two right triangles that have one pair of congruent legs and congruent hypotenuses are congruent (The Hypotenuse-Leg Theorem)?

Both of the triangles are right triangles.

$\angle B$ and $\angle E$ are right angles.

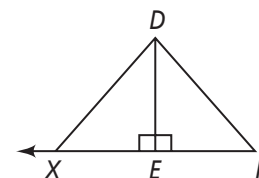
$\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.



How can you prove that $\triangle ABC \cong \triangle DEF$?

Look at $\triangle DEF$. Draw a ray starting at F that passes through E . Mark a point X so that $EX = BC$. Then draw \overline{DX} to create $\triangle DEX$.

See that $\overline{EX} \cong \overline{BC}$. (You drew this.) $\angle DEX \cong \angle ABC$. (All right angles are congruent.) $\overline{DE} \cong \overline{AB}$. (This was given.) So, by SAS, $\triangle ABC \cong \triangle DEX$.



$\overline{DX} \cong \overline{AC}$ (by CPCTC) and $\overline{AC} \cong \overline{DF}$. (This was given.) So, by the Transitive Property of Congruence, $\overline{DX} \cong \overline{DF}$. Then, $\angle DEX \cong \angle DEF$. (All right angles are congruent.) By the Isosceles Theorem, $\angle X \cong \angle F$. So, by AAS, $\triangle DEX \cong \triangle DEF$.

Therefore, by the Transitive Property of Congruence, $\triangle ABC \cong \triangle DEF$.

Problem

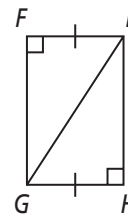
Are the given triangles congruent by the Hypotenuse-Leg Theorem?

If so, write the triangle congruence statement.

$\angle F$ and $\angle H$ are both right angles, so the triangles are both right.

$\overline{GI} \cong \overline{IG}$ by the Reflexive Property and $\overline{FI} \cong \overline{HG}$ is given.

So, $\triangle FIG \cong \triangle HGI$.

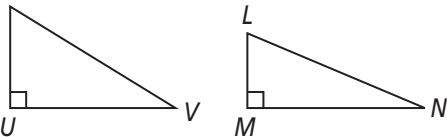


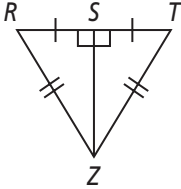
4-6 **Reteaching** (continued)

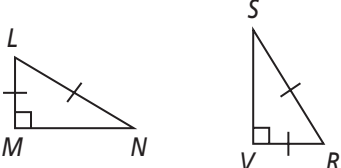
Congruence in Right Triangles

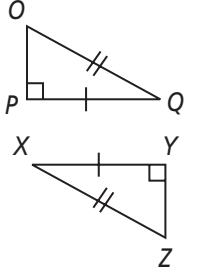
Exercises

Determine if the given triangles are congruent by the Hypotenuse-Leg Theorem. If so, write the triangle congruence statement.

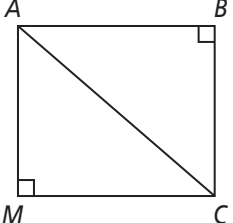
1. 
not congruent

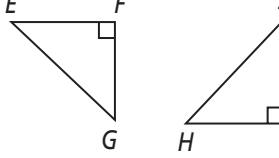
2. 
 $\triangle RSZ \cong \triangle TSZ$

3. 
 $\triangle LMN \cong \triangle RVS$

4. 
 $\triangle OPQ \cong \triangle ZYX$

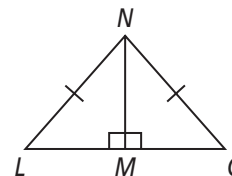
Measure the hypotenuse and length of the legs of the given triangles with a ruler to determine if the triangles are congruent. If so, write the triangle congruence statement.

5. 
 $\triangle ABC \cong \triangle CMA$

6. 
 $\triangle EFG \cong \triangle HIJ$

7. Explain why $\triangle LMN \cong \triangle OMN$. Use the Hypotenuse-Leg Theorem.

Because $\angle NML$ and $\angle NMO$ are right angles, both triangles are right triangles. It is given that their hypotenuses are congruent. Because they share a leg, one pair of corresponding legs is congruent. All criteria are met for the triangles to be congruent by the Hypotenuse-Leg Theorem.



8. Visualize $\triangle ABC$ and $\triangle DEF$, where $AB = EF$ and $CA = FD$. What else must be true about these two triangles to prove that the triangles are congruent using the Hypotenuse-Leg Theorem? Write a congruence statement.

$\angle B$ and $\angle E$ are right angles, or $\angle C$ and $\angle D$ are right angles. $\triangle ABC \cong \triangle DEF$ or $\triangle ABC \cong \triangle FED$.

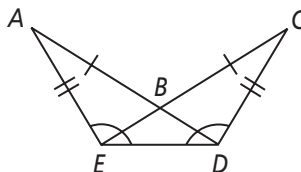
4-7 Reteaching

Congruence in Overlapping Triangles

Sometimes you can prove one pair of triangles congruent and then use corresponding parts of those triangles to prove another pair congruent. Often the triangles overlap.

Problem

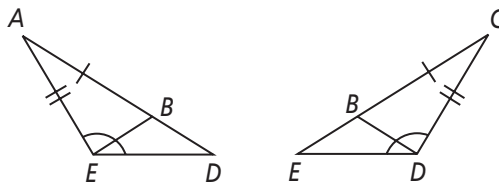
Given: $\overline{AB} \cong \overline{CB}$,
 $\overline{AE} \cong \overline{CD}$,
 $\angle AED \cong \angle CDE$



Prove: $\triangle ABE \cong \triangle CBD$

Think about a plan for the proof. Examine the triangles you are trying to prove congruent. Two pairs of sides are congruent. If the included angles, $\angle A$ and $\angle C$, were congruent, then the triangles would be congruent by SAS.

If the overlapping triangles $\triangle AED$ and $\triangle CDE$ were congruent, then the angles would be congruent by corresponding parts. When triangles overlap, sometimes it is easier to visualize if you redraw the triangles separately.



Now use the plan to write a proof.

Given: $\overline{AB} \cong \overline{CB}$, $\overline{AE} \cong \overline{CD}$, $\angle AED \cong \angle CDE$

Prove: $\triangle ABE \cong \triangle CBD$

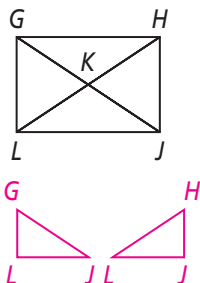
Statements	Reasons
1) $\overline{AE} \cong \overline{CD}$, $\angle AED \cong \angle CDE$	1) Given
2) $\overline{ED} \cong \overline{ED}$	2) Reflexive Property of \cong
3) $\triangle AED \cong \triangle CDE$	3) SAS
4) $\angle A \cong \angle C$	4) CPCTC
5) $\overline{AB} \cong \overline{CB}$	5) Given
6) $\triangle ABE \cong \triangle CBD$	6) SAS

4-7 Reteaching (continued)

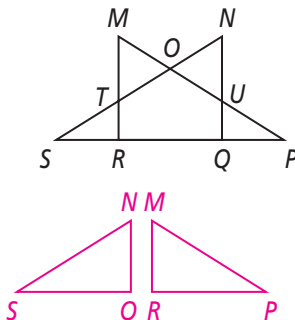
Congruence in Overlapping Triangles

Separate and redraw the overlapping triangles. Identify the vertices.

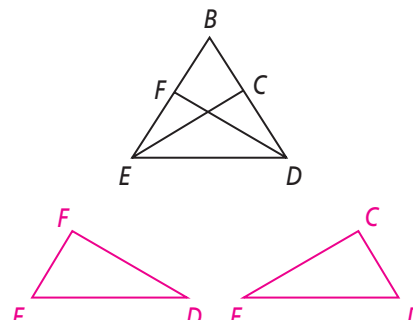
1. $\triangle GLJ$ and $\triangle HJL$



2. $\triangle MRP$ and $\triangle NQS$



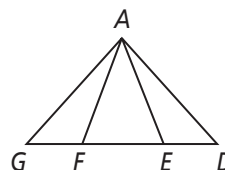
3. $\triangle FED$ and $\triangle CDE$



Fill in the blanks for the two-column proof.

4. **Given:** $\angle AEG \cong \angle AFD$, $\overline{AE} \cong \overline{AF}$, $\overline{GE} \cong \overline{FD}$

Prove: $\triangle AFG \cong \triangle AED$



Statements	Reasons
1) $\angle AEG \cong \angle AFD$, $\overline{AE} \cong \overline{AF}$, $\overline{GE} \cong \overline{FD}$	1) ? Given
2) ? $\triangle AEG \cong \triangle AFD$	2) SAS
3) $\overline{AG} \cong \overline{AD}$, $\angle G \cong \angle D$	3) ? CPCTC
4) ? $\overline{GE} \cong \overline{FD}$	4) Given
5) $GE = FD$	5) ? Def. of \cong
6) $GF + FE = GE$, $FE + ED = FD$	6) ? Seg. Addition Post.
7) $GF + FE = FE + ED$	7) ? Substitution Property
8) ? $GF = ED$	8) Subtr. Prop. of Equality
9) ? $\triangle AFG \cong \triangle AED$	9) ? SAS

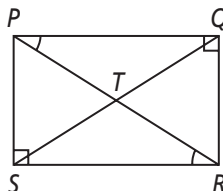
Use the plan to write a two-column proof.

5. **Given:** $\angle PSR$ and $\angle PQR$ are right angles, $\angle QPR \cong \angle SRP$.

Prove: $\triangle STR \cong \triangle QTP$

Plan for Proof:

Prove $\triangle QPR \cong \triangle SRP$ by AAS. Then use CPCTC and vertical angles to prove $\triangle STR \cong \triangle QTP$ by AAS.



Statements: 1) $\angle PSR$ and $\angle PQR$ are rt. \sphericalangle s;
 $\angle QPR \cong \angle SRP$;
 2) $\angle PSR \cong \angle RQP$;
 3) $\overline{PR} \cong \overline{RP}$;
 4) $\triangle QPR \cong \triangle SRP$;
 5) $\angle STR \cong \angle QTP$;
 6) $\overline{PQ} \cong \overline{RS}$;
Reasons: 1) Given; 2) Rt. \sphericalangle are congruent; 3) Refl. Prop. of \cong ; 4) AAS; 5) Vert. \sphericalangle are \cong ; 6) CPCTC; 7) AAS